PHYSICAL REVIEW E VOLUME 55, NUMBER 5 MAY 1997

# Linear modes in the rotating neutron star polar-cap electron-positron plasma

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(Received 17 September 1996)

Linear modes in the rotating neutron star polar-cap electron-positron plasma are considered. Due to the like dynamical behavior among species, linear modes in electron-positron plasmas have some specific features. Rotation of a neutron star generates additional effects on the plasma in the polar-cap region. These effects are different for waves propagating at different angles to the ambient magnetic field. [S1063-651X(97)06105-9]

PACS number(s): 52.60.+h, 52.25.−b, 52.35.Mw, 52.35.Sb

## I. INTRODUCTION

Pulsars are the source of pulsed cosmic radio emission. They were discovered in 1967 and almost immediately associated with rotating neutron stars. Such stars originate from the catastrophic gravitational collapse of ordinary stars of some specific mass, which have exhausted their nuclear fuel. In neutron stars, the gravitational forces are brought to equilibrium by the pressure of strongly compressed neutron matter. Although the mass of the stars is on the order of solar mass, their radius is only about 10 km. The density arises on the order of $10^{14}$ gm/cm$^3$ and conservation of magnetic flux creates a superstrong magnetic field on the order of $10^{16}$ gauss. On the other hand, conservation of angular momentum makes the neutron star a highly rotating object.

The most remarkable property of radio pulsars is the very high stability of the frequency of succession of radiation pulses. It is natural to assume that the high coherence and directivity of observed pulsar radio emission is due to the presence of a magnetic field and plasma in the vicinity of the neutron star (i.e., pulsar magnetosphere) in which the emission is generated. It is suggested [1–3] that cascade generation of $ee^+$ plasma occurs in the polar-cap region of the rotating neutron star. The process of electron-positron pair creation and annihilation also occurs in relativistic plasma at high temperatures. The pair production, which is one of the most effective means of producing an electron-positron plasma, involves high-energy processes under extreme astrophysical conditions, such as solar flares, pulsars, black holes, the jet phenomena associated with active galactic nuclei [4], and in the early universe [5].

In this paper, we study the linear modes in the rotating and strongly magnetized electron-positron plasmas. We investigate the rotational effect on the normal modes in a magnetized plasma. We know that the rotational axis and the magnetic axis are not aligned; therefore, the inclination angle also plays a vital role in the pulsar-magnetosphere plasmas.

## II. BASIC EQUATIONS

We consider two fluid magnetohydrodynamic (MHD) equations to describe the rotating $ee^+$ plasma in the polar-cap region of the neutron star. The equations are the usual continuity and momentum equations for each species, supplemented by Maxwell's equations. In the momentum balance equation, we introduce the Coriolis force $2m(\vec{\omega}_0 \times \vec{v}_s)$ for plasma rotation with rotation frequency $\vec{\omega}_0$. We also introduce a phenomenological damping force $m\nu\vec{v}_s$ proportional to the mass and velocity of the charge and to an effective damping frequency $\nu$. We assume that pressure $P_s(s=e,e^+)$ is isotropic and satisfies $P_s=n_sT_s$, where $n_s$ is the number density of species $s$, and $T_s$ is the species temperature. With the rotation we only consider the Coriolis force, neglecting the centrifugal term, as plasma is considered to be very close to the rotational axis. In the Maxwell system, the rotational effects come through the current; therefore, modification of the field equations is not considered, since the plasma rotation is much smaller than the cyclotron frequency.

The equations considered are, therefore,

$$\frac{\partial n_s}{\partial t}+\vec{\nabla}\cdot n_s\vec{v}_s=0, \tag{1}$$

$$(\partial_t+\vec{v}_s\cdot\vec{\nabla})\vec{v}_s=\frac{q_s}{m}\vec{E}+\frac{q_s}{mc}(\vec{v}_s\times\vec{B}_0)+2(\vec{\omega}_0\times\vec{v}_s)+\nu\vec{v}_s-\frac{T_s}{mn_s}\vec{\nabla}n_s, \tag{2}$$

$$\vec{\nabla}\times\vec{B}=\frac{4\pi}{c}\sum_s q_sn_s\vec{v}_s+\frac{1}{c}\frac{\partial\vec{E}}{\partial t}, \tag{3}$$

$$\vec{\nabla}\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{B}}{\partial t}, \tag{4}$$

$$\vec{\nabla}\cdot\vec{E}=4\pi\sum_s q_sn_s, \tag{5}$$

$$\vec{\nabla}\cdot\vec{B}=0, \tag{6}$$

where $q_s$ denotes the charge ($-e$ or $e$ for $s=e$, $e^+$ respectively), $\vec{v}_s$ the fluid velocity, $c$ the speed of light, $m$ the electron-positron mass, and $\vec{E}$, $\vec{B}$ the electric and magnetic fields, respectively.

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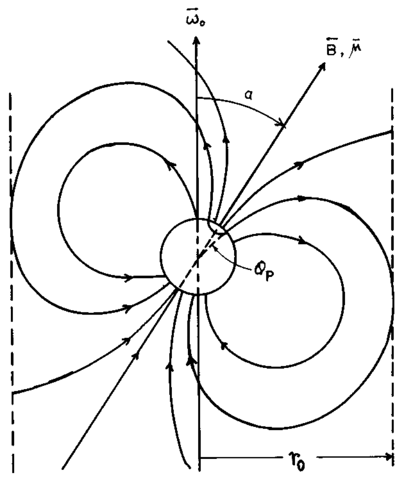


FIG. 1. Schematic view of a magnetized neutron star with a dipole moment $\vec{\mu}$ whose axis makes an angle $\alpha$ with the rotation axis $\vec{\omega}_0$, the closed magnetic field lines $\vec{B}$ extending out to a radius $r_0$ representing either the light cyllinder (isolated neutron star) or the Alfven surface (in the accretion flow of a companion). $\theta_p$ is the polar-cap angle; lines inside $\theta_p$ will be open and reach outside the light cylinder.

## III. DIELECTRIC TENSOR FOR THE ROTATING $ee^+$ PLASMA

We consider the neutron star to be an inclined rotator, as shown in Fig. 1. The magnetic axis makes an inclination angle $\alpha$ with the rotation axis. The highly magnetized $ee^+$ plasma generated in the polar-cap region also rotates with the star within the light cylinder. For a plane-wave propagation of the form

$$e^{i(\vec{k}\vec{r}-\omega t)}, \tag{7}$$

from Eqs. (1) and (2), we find

$$\vec{v}_s=\frac{1}{\Omega_{0s}'^2-\omega^2\lambda}\left[\frac{q_s}{m}\left\{-i\omega\lambda\vec{E}+\frac{i\vec{\Omega}_{0s}'}{\omega\lambda}(\vec{E}\cdot\vec{\Omega}_{0s}')+\vec{E}\times\vec{\Omega}_{0s}'\right\}\right.$$
$$\left.-v_{ts}^2\,\frac{\vec{k}\cdot\vec{v}_s}{\omega}\left\{\omega\lambda\vec{k}+i\vec{k}\times\vec{\Omega}_{0s}'-\frac{\vec{\Omega}'}{\omega\lambda}(\vec{k}\cdot\vec{\Omega}_{0s}')\right\}\right], \tag{8}$$

where

$$\lambda=1+i\nu/\omega, \quad v_{ts}^2=\frac{T_s}{m},$$

$$\vec{\Omega}_{0s}'=\vec{\Omega}_{0s}-2\vec{\omega}_0, \quad \vec{\Omega}_{0s}=\frac{q_s\vec{B}_0}{mc}.$$

For simplicity, we consider the magnetic field in the polar cap region to be along the $\vec{z}$ axis, i.e., $\vec{B}_0=B_0\vec{z}$, and the wave propagation vector $\vec{k}$ to be on the $\hat{x},\hat{z}$ plane, i.e., $\vec{k}=\{k_x,0,k_z\}$. We also consider the rotational frequency $\vec{\omega}_0=\{\omega_{0x},0,\omega_{0z}\}$ where $\omega_{0x}=\omega_0\sin\alpha$, $\omega_{0z}=\omega_0\cos\alpha$. Then, from Eq. (8) one can easily find particle velocities along the $\hat{x}$ and $\hat{z}$ directions:

$$v_{sx}=\frac{q_s}{m}\,\frac{1}{\Omega_{0s}'^2-\omega^2\lambda^2+k^2v_{ts}^2\lambda\{1-[(\vec{k}\cdot\vec{\Omega}_{0s}')^2/(k^2\omega^2\lambda^2)]\}}$$
$$\times\left[i\left\{\left(-\omega\lambda+\frac{\Omega_{0sx}'^2}{\omega\lambda}\right)+\frac{k_z^2v_{ts}^2}{\omega}\right\}E_x\right.$$
$$+\left\{\Omega_{0sz}'-\frac{k_zv_{ts}^2}{\omega^2\lambda}(\vec{k}\cdot\vec{\Omega}_{0s}')\right\}E_y$$
$$\left.+i\left\{\frac{\Omega_{0sx}'\Omega_{0sz}'}{\omega\lambda}-\frac{k_zk_xv_{ts}^2}{\omega}\right\}E_x\right], \tag{9}$$

$$v_{sz}=\frac{q_s}{m}\,\frac{1}{\Omega_{0s}'^2-\omega^2\lambda^2+k^2v_{ts}^2\lambda\{1-[(\vec{k}\cdot\vec{\Omega}_{0s}')^2/(k^2\omega^2\lambda^2)]\}}$$
$$\times\left[i\left\{\frac{\Omega_{0sx}'\Omega_{0sz}'}{\omega\lambda}-\frac{k_zk_xv_{ts}^2}{\omega}\right\}E_x\right.$$
$$-\left\{\Omega_{0sx}'-\frac{k_xv_{ts}^2}{\omega^2\lambda}(\vec{k}\cdot\vec{\Omega}_{0s}')\right\}E_y$$
$$\left.+i\left\{\left(-\omega\lambda+\frac{\Omega_{0sz}'^2}{\omega\lambda}\right)+\frac{k_x^2v_{ts}^2}{\omega}\right\}E_z\right], \tag{10}$$

and for $k_y=0$, from equation of motion (2), we easily get

$$v_{sy}=\frac{q_s}{m}\,\frac{1}{\Omega_{0s}'^2-\omega^2\lambda}(-\Omega_{0sz}'E_x-i\omega\lambda E_y+\Omega_{0sx}'E_z). \tag{11}$$

From the relation

$$\vec{D}=\epsilon_{ij}E_j=\vec{E}+\frac{4\pi i}{\omega}\sum_s q_sn_s\vec{v}_s, \tag{12}$$

with $\vec{v}_s$ defined by Eq. (9)–(11), we determine the dielectric tensor for the rotating magnetized plasma to be

$$\epsilon_{ij}=\begin{pmatrix}\epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz}\\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz}\\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz}\end{pmatrix}, \tag{13}$$

with

$$\epsilon_{xx}=1-\sum_s\frac{\omega_{ps}^2}{\omega^2\lambda}$$
$$\times\frac{\omega^2\lambda^2-\Omega_{0sx}'^2-\lambda k_z^2v_{ts}^2}{\omega^2\lambda^2-\Omega_{0s}'^2-k^2v_{ts}^2\lambda\{1-[(\vec{k}\cdot\vec{\Omega}_{0s}')^2/(k^2\omega^2\lambda^2)]\}},$$

$$\epsilon_{xy}=-i\sum_s \frac{\omega_{ps}^2}{\omega\lambda}\times\frac{\Omega'_{0sz}-\frac{k_z v_{ts}^2}{\omega^2\lambda}\vec{k}\cdot\vec{\Omega}'_{0s}}{\omega^2\lambda^2-\Omega'^2_{0s}-k^2v_{ts}^2\lambda\{1-[(\vec{k}\cdot\vec{\Omega}'_{0s})^2/(k^2\omega^2\lambda^2)]\}},$$

$$\epsilon_{xz}=\sum_s \frac{\omega_{ps}^2}{\omega^2\lambda}\times\frac{\Omega'_{0sx}\Omega'_{0sz}-\lambda k_z k_x v_{ts}^2}{\omega^2\lambda^2-\Omega'^2_{0s}-k^2v_{ts}^2\lambda\{1-[(\vec{k}\cdot\vec{\Omega}'_{0s})^2/(k^2\omega^2\lambda^2)]\}},$$

$$\epsilon_{yx}=i\sum_s \frac{\omega_{ps}^2\Omega'_{0sz}}{\omega(\omega^2\lambda^2-\Omega_{0sz}^2)},$$

$$\epsilon_{yy}=1-\sum_s \frac{\omega_{ps}^2\lambda}{\omega^2\lambda^2-\Omega'^2_{0s}},$$

$$\epsilon_{yz}=-i\sum_s \frac{\omega_{ps}^2\Omega'_{0sx}}{\omega(\omega^2\lambda^2-\Omega_{0s}^2)},$$

$$\epsilon_{zx}=\epsilon_{xz},$$

$$\epsilon_{zy}=i\sum_s \frac{\omega_{ps}^2}{\omega\lambda}\times\frac{\Omega'_{0sx}-[(k_x v_{ts}^2)/(\omega^2\lambda)]\vec{k}\cdot\vec{\Omega}'_{0s}}{\omega^2\lambda^2-\Omega'^2_{0s}-k^2v_{ts}^2\lambda\{1-[(\vec{k}\cdot\vec{\Omega}'_{0s})^2/(k^2\omega^2\lambda^2)]\}},$$

$$\epsilon_{zz}=1-\sum_s \frac{\omega_{ps}^2}{\omega^2\lambda}\times\frac{\omega^2\lambda^2-\Omega_{0sz}^2-\lambda k_x^2 v_{ts}^2}{\omega^2\lambda^2-\Omega'^2_{0s}-k^2v_{ts}^2\lambda\{1-[(\vec{k}\cdot\vec{\Omega}'_{0s})^2/(k^2\omega^2\lambda^2)]\}}. \tag{14}$$

The obtained dielectric tensor is more simplified for electron-positron plasma near the polar region. As the neutron star magnetic field is superstrong ($\sim 10^{12}$ gauss), we consider the cyclotron frequency to be much higher than the plasma rotation, i.e., $|\vec{\Omega}_{0s}|^2 \gg |\vec{\omega}_0|^2$. So we assume that $|\vec{\Omega}_{0s}|^2 \simeq \omega_c^2$ and $(\vec{k}\cdot\vec{\Omega}_{0s})^2 \simeq k_z^2\omega_c^2$ [where $\omega_c=(eB_0)/mc$] and we neglect the damping effect, i.e., $\lambda=1$. Thus for $ee^+$ plasma near the polar cap of the neutron star, we have

$$\epsilon_{xx}=1-\frac{\omega_p^2}{\omega^2}\frac{\omega^2-k_z^2v_t^2-4\omega_{0x}^2}{\omega^2-\omega_c^2-k^2v_t\{1-[(k_z^2\omega_c^2)/(k^2\omega^2)]\}}$$

$$\epsilon_{xy}=i\,\frac{\omega_p^2}{\omega}\frac{2[1-(k_z^2v_t^2)/\omega^2]\omega_{0z}-[(2k_xk_zv_t^2)/\omega^2]\omega_{0x}}{\omega^2-\omega_c^2-k^2v_t^2\{1-[(k_z^2\omega_c^2)/(k^2\omega^2)]\}}$$

$$\epsilon_{xz}=\frac{\omega_p^2}{\omega^2}\frac{4\omega_{0x}\omega_{0z}-k_xk_zv_t^2}{\omega^2-\omega_c^2-k^2v_t^2\{1-[(k_z^2\omega_c^2)/(k^2\omega^2)]\}}$$

$$\epsilon_{yx}=-i\,\frac{\omega_p^2}{\omega}\frac{2\omega_{0z}}{\omega^2-\omega_c^2}$$

$$\epsilon_{yy}=1-\frac{\omega_p^2}{\omega^2-\omega_c^2}$$

$$\epsilon_{yz}=i\,\frac{\omega_p^2}{\omega}\frac{2\omega_{0x}}{\omega^2-\omega_c^2}$$

$$\epsilon_{zx}=\epsilon_{xz}=\frac{\omega_p^2}{\omega^2}\frac{4\omega_{0x}\omega_{0z}-k_xk_zv_t^2}{\omega^2-\omega_c^2-k^2v_t^2\{1-[(k_z^2\omega_c^2)/(k^2\omega^2)]\}}$$

$$\epsilon_{zy}=i\,\frac{\omega_p^2}{\omega}\frac{2[1-(k_x^2v_t^2)/\omega^2]\omega_{0x}-[(2k_xk_zv_t^2)/\omega^2]\omega_{0z}}{\omega^2-\omega_c^2-k^2v_t^2\{1-[(k_z^2\omega_c^2)/(k^2\omega^2)]\}}$$

$$\epsilon_{zz}=1-\frac{\omega_p^2}{\omega^2}\frac{\omega^2-\omega_c^2-k_x^2v_t^2}{\omega^2-\omega_c^2-k^2v_t^2\{1-[(k_z^2\omega_c^2)/(k^2\omega^2)]\}}. \tag{15}$$

Here, $\omega_p^2=(8\pi e^2 n_0)/m$ and $v_t^2=T/m$. As plasma is considered to be isotropic, so $T_e=T_p=T$ is considered. From Maxwell equations (3)–(6), we have the wave equation

$$\left\{k_ik_j-k^2\delta_{ij}+\frac{\omega^2}{c^2}\,\epsilon_{ij}(\omega,k)\right\}E_j=0. \tag{16}$$

Let $k_x=k\ \sin\theta$, $k_z=k\ \cos\theta$, where $\theta$ is the angle between the wave vector $\vec{k}$ and the magnetic field $\vec{B}_0$. Then, defining $N^2=(k^2c^2)/(\omega^2)$, we have the wave equations

$$\begin{pmatrix}\epsilon_{xx}-N^2\cos^2\theta & \epsilon_{xy} & \epsilon_{xz}+N^2\ \sin\theta\ \cos\theta\\ \epsilon_{yx} & \epsilon_{yy}-N^2 & \epsilon_{yz}\\ \epsilon_{zx}+N^2\sin\theta\ \cos\theta & \epsilon_{zy} & \epsilon_{zz}-N^2\sin\theta\end{pmatrix}\times\begin{pmatrix}E_x\\E_y\\E_z\end{pmatrix}=0. \tag{17}$$

## IV. WAVE PROPAGATION PARALLEL TO THE MAGNETIC FIELD

First we consider the simplest case of wave propagation along $B_0\hat{z}$. So in this case, $\theta=0$ (i.e., $k_x=0$). Let us consider that the inclination angle $\alpha$ is very small (i.e., $\omega_{0x}\sim 0$). In this particular case the wave equation (17) may be written as

$$\begin{pmatrix}\epsilon+\chi-N^2 & ig & 0\\ -ig & \epsilon-N^2 & 0\\ 0 & 0 & \eta\end{pmatrix}\begin{pmatrix}E_x\\E_y\\E_z\end{pmatrix}=0, \tag{18}$$

where

$$\epsilon=1-\frac{\omega_p^2}{\omega^2-\omega_c^2},$$

$$\chi = \frac{4\omega_{0x}^2\omega_p^2}{(\omega^2-\omega_c^2)(\omega^2-k^2v_t^2)},$$

$$g = \frac{2\omega_{0z}\omega_p^2}{\omega(\omega^2-\omega_c^2)},$$

$$\eta = 1 - \frac{\omega_p^2}{\omega^2-k^2v_t^2}.$$

### A. The Langmuir mode

The mode $\eta = 0 = 1 - [\omega_p^2/(\omega^2-k^2v_t^2)]$ represents the electrostatic Langmuir wave with $E_z \neq 0$ along the magnetic field. The dispersion for the plasma mode is the usual relation

$$\omega(k) = \omega_p(1+\tfrac{3}{2}k^2\lambda_D^2), \tag{19}$$

where we assume that $v_t^2/\omega_p^2 = (T/m)/\omega_p^2 = 3\lambda_D^2$ and $k^2\lambda_D^2 \ll 1$. Here we see that plasma rotation does not have a significant effect on Langmuir waves along the magnetic field.

### B. The transverse mode

The dispersion relation for the transverse mode along the magnetic field is described by the equation

$$(\epsilon+\chi-N^2)(\epsilon-N^2)-g^2=0. \tag{20}$$

If we define

$$\epsilon_r = \epsilon+\chi+g, \qquad \epsilon_l = \epsilon - g, \tag{21}$$

the dispersion relation (20) can be written as

$$(N_r^2-\epsilon_r)(N_l^2-\epsilon_l) = \chi g. \tag{22}$$

The right-hand side of Eq. (22) is related to the rotation ($\omega_0$) of the plasma and to the inclination ($\alpha$) of the rotating neutron star. These two features have significant effects on the dispersion of transverse plasma modes propagating along the magnetic field. For simplicity, let us consider the modes very close to the polar axis (i.e., $\alpha \sim 0$, which gives $\omega_{0x} \sim 0$ and $\omega_{0z} \sim \omega_0$). In this case, from Eq. (22) we find that

$$N_r^2 = \epsilon_r = 1 - \frac{\omega_p^2}{\omega^2-\omega_c^2}\left(1-\frac{2\omega_0}{\omega}\right), \tag{23}$$

$$N_l^2 = \epsilon_l = 1 - \frac{\omega_p^2}{\omega^2-\omega_c^2}\left(1+\frac{2\omega_0}{\omega}\right). \tag{24}$$

We see that, without plasma rotation ($\omega_0=0$), one may easily recover the earlier results [6–8],

$$N_1^2 = N_2^2 = 1 - \frac{\omega_p^2}{\omega^2-\omega_c^2}, \tag{25}$$

which describes that, in electron-positron plasma, both the right-hand and left-hand circularly polarized waves are described by the same dispersion relation and both electron and positron can resonate with the wave if $\omega \to \omega_c$. Thus both the modes are extraordinary waves (x modes).

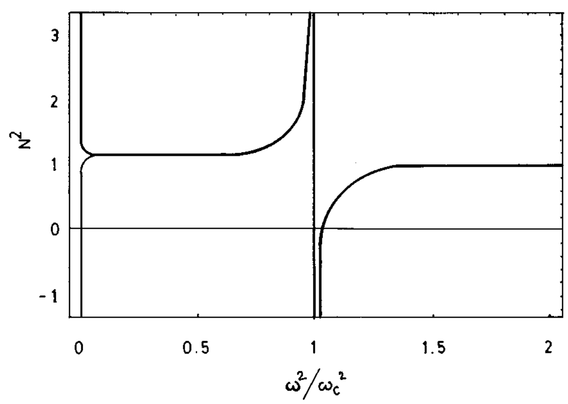


FIG. 2. Refractive indices of electromagnetic waves propagating along the magnetic field in the rotating electron-positron plasma. The dimensionless parameters are $\omega_p/\omega_c = 0.1$, $\omega_0/\omega_c = 0.01$. The thin curve represents the right-polarized wave and the thick curve represents the left-polarized wave.

But, for $\omega_0 \neq 0$, we see that $N_r \neq N_l$. So the waves' refractive indices differ in this case, as shown in Fig. 2.

Now, let us investigate both the right-hand (r mode) and left-hand (l mode) polarized waves in the rotating plasma. For this, we rewrite Eqs. (23) and (24) as

$$\frac{k^2c^2}{\omega^2} = 1 - \frac{\tilde{\omega}_p{}^2}{\omega^2-\omega_c^2}, \tag{26}$$

where

$$\tilde{\omega}_p{}^2 = \begin{cases} \omega_p^2\left(1-\dfrac{2\omega_0}{\omega}\right), & \text{for r mode,} \\ \omega_p^0\left(1+\dfrac{2\omega_0}{\omega}\right), & \text{for l mode.} \end{cases}$$

Then from Eq. (26) we find

$$\omega^2 = \tfrac{1}{2}\left[k^2c^2+\tilde{\omega}_p{}^2+\omega_c^2 \pm \sqrt{(k^2c^2+\tilde{\omega}_p{}^2+\omega_c^2)^2-4k^2c^2\omega_c^2}\right]. \tag{27}$$

#### *1. High-frequency modes*

Equation (27) describes two branches of modes, high-frequency (+ sign) and low-frequency (− sign) waves. First, we consider the plus sign. Then, in the long-wavelength limit $(k \to 0)$ we find

$$\omega(k) = (\omega_p{}^2+\omega_c^2)^{1/2}\left[1-\frac{k^2c^2\omega_c^2}{2(\omega_p{}^2+\omega_c^2)^2}+\cdots\right], \tag{28}$$

while in the short-wavelength limit $(k \to \infty)$ we have

$$\omega(k) = kc\left(1+\frac{1}{2}\frac{\tilde{\omega}_p{}^2}{k^2c^2}+\cdots\right). \tag{29}$$

It should be noted that for high-frequency waves $(\omega_0/\omega) \ll 1$, so $\tilde{\omega}_p^2 = \omega_p^2$ in this case. Thus, plasma rotation does not significantly affect the waves in the upper branch.

### 2. Low-frequency modes

Now we consider the low-frequency branch, taking the minus sign in Eq. (27). First, we do an analysis in the long-wavelength limit. For $k\to 0$, we find

$$\omega(\vec{k})=\begin{cases}\dfrac{\sqrt{\omega_0^2+[1+(v_A^2/c^2)]k^2v_A^2}+\omega_0}{1+(v_A^2/c^2)} & \text{for r mode,}\\[2ex] \dfrac{\sqrt{\omega_0{}^2+[1+(v_A^2/c^2)]k^2v_A^2}-\omega_0}{1+(v_A^2/c^2)}, & \text{for l mode.}\end{cases} \tag{30}$$

For $\omega_0=0$ (no rotation), we get the usual low-frequency branch

$$\omega(k)=\frac{kc}{\sqrt{1+(c^2/v_A^2)}}. \tag{31}$$

But for plasma rotation $(\omega_0\neq 0)$, we see that the modes are different. In the long-wavelength limit, for $k=0$, we find

$$\omega=\begin{cases}\dfrac{2\omega_0}{1+(v_A^2/c^2)} & \text{for r mode,}\\ 0 \quad \text{for l mode,}\end{cases} \tag{32}$$

which shows that in the rotating plasma a right-polarized long wave and an extremely low-frequency mode exist.

For a dense plasma $(\omega_p^2\gg\omega_c^2)$, we have $v_A^2/c^2\ll 1$. So, from Eq. (30) we find

$$\omega(k)=\begin{cases}\sqrt{\omega_0^2+k^2v_A^2}+\omega_0, & \text{for r mode,}\\ \sqrt{\omega_0^2+k^2v_A^2}-\omega_0, & \text{for l mode}\end{cases}. \tag{33}$$

In the short-wavelength limit, i.e., $k\to\infty$, we find

$$\omega=\omega_c, \tag{34}$$

which shows that, for larger $k$, both modes behave similarly and plasma rotation does not have a significant effect.

The dispersion relation ($\omega$ vs $k$) is schematically shown in Fig. 3. It shows that high-frequency waves are not influenced very much by plasma rotation but the low-frequency mode with a larger wavelength splits up into two branches.

Thus, we perform a simple analysis of modes propagating along the magnetic field, close to the polar axis in a rotating pulsar plasma. Obviously, it is useful to outline the main features of a rotating plasma with those in the nonrotating plasma. One finds the following features in the rotating electron-positron polar-cap plasma.

(1) The Langmuir mode is not affected by plasma rotation.

(2) Both the right and left circularly polarized waves have resonances near the cyclotron frequency.

(3) For high-frequency waves, plasma rotation has no significant effect on wave propagation.

(4) For low-frequency modes in the long-wavelength regime, plasma rotation plays a significant role. The plasma dispersion relation for right-circularly-polarized waves differs from that of left-circularly-polarized waves.

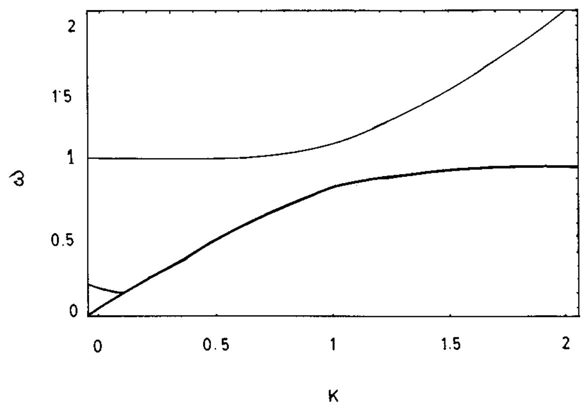


FIG. 3. The dispersion relation for the transverse collective mode propagating parallel to the magnetic field. In the high-frequency regime (thin curve) both the right- and the left-polarized waves have the same dispersion. But in the low-frequency regime (thick curve), the right- and left-polarized waves split for smaller $k$.

(5) A long-wavelength low-frequency transverse mode with the frequency close to plasma rotation may propagate along the magnetic field.

## V. WAVE PROPAGATION PERPENDICULAR TO THE MAGNETIC FIELD

Now we consider the wave propagation perpendicular to the magnetic field. We assume $\vec{k}=\{k,0,0\}$. Then for $e,e^+$ plasma close to the polar axis (i.e., $\alpha\to 0$ or $\omega_{0x}\to 0$), the wave equations may be written as

$$\begin{pmatrix}\epsilon_1 & ig_1 & 0\\ -ig_2 & \epsilon_2-N^2 & 0\\ 0 & 0 & \eta_1-N^2\end{pmatrix}\begin{pmatrix}E_x\\ E_y\\ E_z\end{pmatrix}=0, \tag{35}$$

where

$$\epsilon_1=1-\frac{\omega_p^2}{\omega^2-\omega_c^2-k^2v_t^2},$$

$$\epsilon_2=1-\frac{\omega_p^2}{\omega^2-\omega_c^2},$$

$$g_1=\frac{2\omega_0}{\omega}\frac{\omega_p^2}{\omega^2-\omega_c^2-k^2v_t^2},$$

$$g_2=\frac{2\omega_0}{\omega}\frac{\omega_p^2}{\omega^2-\omega_c^2},$$

$$\eta_1=1-\frac{\omega_p^2}{\omega^2}.$$

$N_1^2=\eta_1=1-(\omega_p^2/\omega^2)$ represents the transverse wave with the electric field parallel to the ambient magnetic field ($\vec{E}\parallel\vec{B}_0$ but $\vec{k}\perp\vec{B}_0$). In this mode, magnetic field has no effect. So this is the usual, ordinary, or l mode.

The other dispersion relation is described by the equation

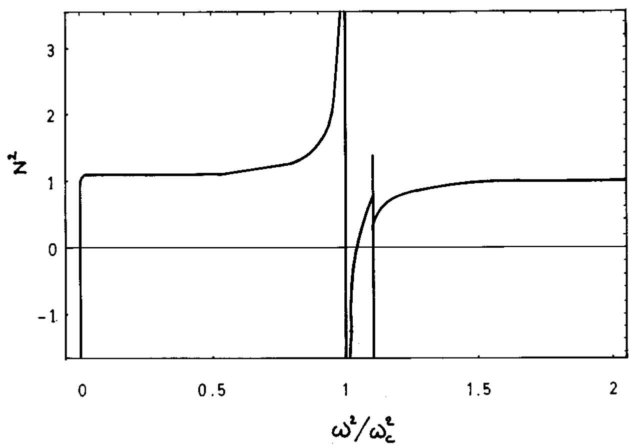


FIG. 4. Refractive indices of electromagnetic waves propagating perpendicular to the rotating electron-positron plasma. The values of dimensionless parameters are $\omega_p/\omega_c=0.1$, $\omega_0/\omega_c=0.01$, $(k^2v_t^2)/\omega_c^2=0.001$. With the usual cutoff at $\omega^2=\omega_c^2$ there occurs an extra cutoff at $\omega^2=\omega_c^2+\omega_p^2+k^2v_t^2$ due to plasma rotation.

$$N_2^2=\epsilon_2-\frac{g_1g_2}{\epsilon_1}$$
$$=1-\frac{\omega_p^2}{\omega^2-\omega_c^2}\left(1+\frac{4\omega_0^2}{\omega^2}\frac{\omega_p^2}{\omega^2-\omega_c^2-\omega_p^2-k^2v_t^2}\right). \tag{36}$$

Refractive indices of these modes are shown graphically in Fig. 4.

We see that plasma rotation does not affect the ordinary modes, but it affects the extraordinary mode in the low-frequency regime. With the usual cutoff frequency at $\omega^2=\omega_c^2$, there occurs an extra cutoff at $\omega^2=\omega_c^2+\omega_p^2+k^2v_t^2$ entirely due to the rotation ($\omega_0$) of the plasma. From the first line of Eq. (35), we get the ratio of the longitudinal to the transverse components,

$$\left|\frac{E_x}{E_y}\right|=\left|\frac{g_1}{\epsilon_1}\right|=\left|\frac{2\omega_0}{\omega}\frac{\omega_p^2}{\omega^2-\omega_c^2-\omega_p^2-k^2v_t^2}\right|. \tag{37}$$

In general, rotation of the star is much less than the wave frequency, i.e., $\omega_0/\omega \ll 1$. So, essentially, $E_x\rightarrow 0$, $E_y\neq 0$. Again, for $\omega\gg\omega_c$, the longitudinal component $\omega_p$ is also small. On the other hand, at the frequency

$$\omega=(\omega_c^2+\omega_p^2+k^2v_t^2)^{1/2}, \tag{38}$$

there is a resonance, which implies that $|E_x/E_y|\rightarrow\infty$ or $E_y\rightarrow 0$, i.e., the wave becomes purely longitudinal.

Thus we find some specific features of transverse modes propagating perpendicular to the magnetic field in a rotating electron-positron plasma.

(1) Both the ordinary and extraordinary modes can exist in the rotating electron-positron plasma.

(2) The cutoff frequency for the extraordinary mode is shifted due to the rotation of plasma.

(3) Both the ordinary and extraordinary waves are linearly polarized.

## VI. WAVE PROPAGATION AT AN ARBITRARY ANGLE TO THE MAGNETIC FIELD

To investigate the wave propagation at an arbitrary angle to the magnetic field, it is convenient to consider the direction of wave vector $\vec{k}$ along the $\hat{z}$ direction, i.e., $\vec{k}=\{0,0,k\}$ and $\vec{B}_0=\{B_{0x},0,B_{0z}\}$, where $B_{0x}=B_0\sin\theta$, $B_{0z}=B_0\cos\theta$. For waves propagating very close to the polar axis ($\alpha=0$, which gives $\omega_{0x}=0$) in the cold plasma approximation ($v_t^2=0$), one may obtain the following equations:

$$\begin{pmatrix} N^2-\epsilon_{xx} & -\epsilon_{xy} & -\epsilon_{xz} \\ -\epsilon_{yx} & N^2-\epsilon_{yy} & -\epsilon_{yz} \\ -\epsilon_{zx} & -\epsilon_{zy} & -\epsilon_{zz}\end{pmatrix}\begin{pmatrix}E_x\\E_y\\E_z\end{pmatrix}=0. \tag{39}$$

In the system of axes with $z$ axis parallel to $\vec{k}$, the third line of matrix equation (39) is independent of $N^2$. So the $z$-component of the electric field is dependent on the other two, i.e.,

$$E_z=-\frac{1}{\epsilon_{zz}}(\epsilon_{zx}E_z+\epsilon_{zy}E_y). \tag{40}$$

The remaining two equations are

$$\begin{pmatrix}\eta_{xx}-N^2 & \eta_{xy}\\ \eta_{yx} & \eta_{yy}-N^2\end{pmatrix}\begin{pmatrix}E_x\\E_y\end{pmatrix}=0, \tag{41}$$

where

$$\eta_{xx}=\epsilon_{xx}-\frac{1}{\epsilon_{zz}}\epsilon_{xz}\epsilon_{zx},$$

$$\eta_{xy}=\epsilon_{xy}-\frac{1}{\epsilon_{zz}}\epsilon_{xz}\epsilon_{zy},$$

$$\eta_{yx}=\epsilon_{yx}-\frac{1}{\epsilon_{zz}}\epsilon_{yz}\epsilon_{zx},$$

$$\eta_{yy}=\epsilon_{yy}-\frac{1}{\epsilon_{zz}}\epsilon_{yz}\epsilon_{zy}, \tag{42}$$

with

$$\epsilon_{xx}=\epsilon\cos^2\theta+\eta\sin^2\theta,$$

$$\epsilon_{xy}=-\epsilon_{yx}=ig\cos\theta,$$

$$\epsilon_{xz}=\epsilon_{zx}=(\epsilon-\eta)\sin\theta\cos\theta,$$

$$\epsilon_{yy}=\epsilon,$$

$$\epsilon_{yz}=-\epsilon_{zy}=-ig\sin\theta,$$

$$\epsilon_{zz}=\epsilon\sin^2\theta+\eta\cos^2\theta.$$

The ratio of $|E_x/E_y|$ gives the degree of the ellipticity of the normal modes. With the quantities given in Eq. (42), after some calculation, one may find

$$\left|\frac{E_x}{E_y}\right| = \frac{\eta_{xy}}{N^2-\eta_{xx}} = \frac{i2\omega_0}{\omega}\frac{\omega_p^2}{\omega^2-\omega_c^2}\left(1-\frac{\omega_p^2}{\omega^2}\right)\cos\theta$$

$$\times\frac{1}{(k^2c^2/\omega^2)-\{[(\omega^2-\omega_c^2-\omega_p^2)(\omega^2-\omega_p^2-k^2v_t^2)]/[(\omega^2-k^2v_t^2)(\omega^2-\omega_c^2-\omega_p^2)\sin^2\theta+(\omega^2-\omega_c^2)(\omega^2-\omega_p^2-k^2v_t^2)\cos^2\theta]\}}. \quad (43)$$

## VII. SUMMARY AND DISCUSSION

We have studied the linear modes in a strongly magnetized and rotating electron-positron plasma. The study is related to plasmas in the polar cap region of a rotating neutron star. We know that, in general, electron-positron plasma even in the nonrotational case differs significantly from the usual electron-ion plasma [9]. The following events take place.

(1) Due to the same charge-to-mass ratio of the electron and positron, the dynamical behavior is the same.

(2) Due to the same time scales, in the thermal equilibrium both species have the same temperature, in contrast to the electron-ion plasma, where different temperatures may be maintained.

(3) In the presence of a magnetic field, the electron and positron perform a gyromotion at the same frequency and the plasma couples to the left- and right-circularly-polarized waves equally.

Now, in case of plasma rotation and strong magnetization of the pulsar plasma, we find some specific features of linear modes in the electron-positron plasmas. For waves propagating along the magnetic field we find the following.

(1) Plasma rotation does not have a significant effect on Langmuir and high-frequency transverse modes.

(2) Both the right- and left-circularly-polarized waves have resonances near the cylotron frequency.

(3) For low-frequency modes in the long-wavelength regime, plasma rotation plays a vital role. Plasma dispersion relations for right-circularly-polarized waves differ from that of left-polarized waves.

For waves propagating perpendicular to the magnetic field we find: The following.

(1) Both the ordinary and extraordinary modes can exist even in the rotating electron-positron plasma.

(2) With the usual cutoff there occurs an extra cutoff due to the rotation of the plasma.

(3) Both the ordinary and extraordinary waves are linearly polarized.

Finally, we study the modes propagating at an arbitrary angle to the magnetic field. We derive an expression [Eq. (43)] that describes the ellipticity of the normal modes. We find that the ellipticity is mostly defined by the rotation of the plasma. In the nonrotational plasma it is simply zero, while it increases with the increase of the rotation ($\omega_0$) of the star. The ellipticity of the modes is also related to the angle propagation $\theta$ with respect to the ambient magnetic field. For waves propagating along the magneting field ($\theta=0$), the ellipticity is maximum, while for waves propagating perpendicular to the magnetic field ($\theta=\pi/2$), the ellipticity is zero. This means that waves propagating perpendicular to the magnetic field are linearly polarized.

Thus, the specific features of linear modes in the rotating and strongly magnetized electron-positron plasma in the polar cap region of a neutron star are revealed. More detailed description may be obtained using the kinetic approach. Nonlinear modes are similarly important. We wish to study and report on these cases in the future.

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